Un nuevo algoritmo multiobjetivo con selección de operadores adaptativa basada en lógica difusa:FAME.

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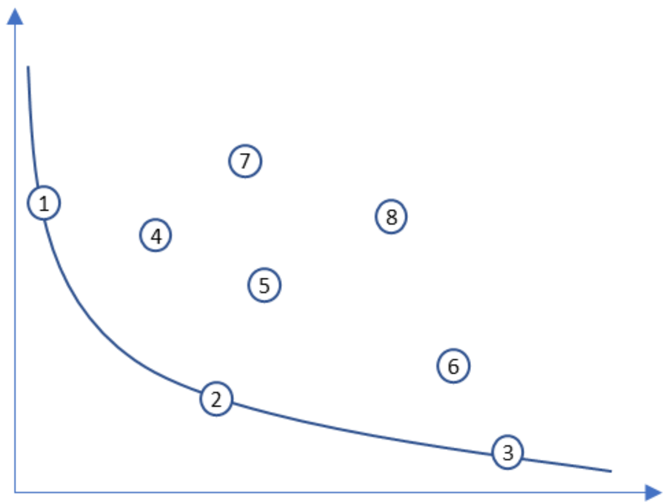
Introducción

Problema MOP.

$\vec{f}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})]$.
Un espacio de soluciones factibles Ω.

Problema MOP:
Encontrar $\vec{x} \in \Omega$ que optimice $\vec{f}(\vec{x})$.



$\vec{\boldsymbol{x}} \prec \vec{\boldsymbol{x}}'$ cuando se cumple:

1) $\vec{x}$ no empeora ningún objetivo respecto a $\vec{x}'$.
2) $\vec{x}$ es estrictamente mejor en al menos un objetivo con respecto a $\vec{x}'$.

ÓPTIMO DE PARETO

$\vec{x}^*$ es óptimo si $\nexists\, \vec{x}' \in \Omega$ tal que $\vec{x}' \prec \vec{x}^*$.

CONJUNTO ÓPTIMO PARETO

Dado un MOP, el conjunto óptimo se define $PS^* = \{\vec{x}^* \in \Omega\}$.

FRENTE ÓPTIMO DE PARETO

Dado un MOP y PS^*, el frente óptimo de Pareto es $PF^* = \left\{\vec{f}(\vec{x}) | \vec{x} \in PS^*\right\}$.

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Instance	Objectives and PSs	Variable Bounds
F1	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2$ where $J_1 = \{j \mid j$ is odd and $2 \le j \le n\}$ and $J_2 = \{j \mid j$ is even and $2 \le j \le n\}$. Its PS is $x_j = x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \dots, n$.	$[0,1]^n$
F2	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2$ where J_1 and J_2 are the same as those of F1. Its PS is $x_j = \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \dots, n$.	$[0,1] \times [-1,1]^{n-1}$
F3	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - 0.8x_1\cos(6\pi x_1 + \frac{j\pi}{n}))^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}))^2$ where J_1 and J_2 are the same as those of F1. Its PS is $x_j = \begin{cases} 0.8x_1\cos(6\pi x_1 + \frac{j\pi}{n}) & j \in J_1 \\ 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}) & j \in J_2 \end{cases}$	$[0,1] \times [-1,1]^{n-1}$
F4	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - 0.8x_1\cos(\frac{6\pi x_1 + \frac{j\pi}{n}}{3}))^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}))^2$ where J_1 and J_2 are the same as those of F1. Its PS is $x_j = \begin{cases} 0.8x_1\cos(\frac{6\pi x_1 + \frac{j\pi}{n}}{3}) & j \in J_1 \\ 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}) & j \in J_2 \end{cases}$	$[0,1] \times [-1,1]^{n-1}$
F5	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}\{x_j - [0.3x_1^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\cos(6\pi x_1 + \frac{j\pi}{n})\}^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}\{x_j - [0.3x_1^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\sin(6\pi x_1 + \frac{j\pi}{n})\}^2$ where J_1 and J_2 are the same as those of F1.	$[0,1] \times [-1,1]^{n-1}$

Instance	Variable Space	Objective Function	Characteristic
GLT1	$[0,1] \times [-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(2 - x_1 - \text{sign}(\cos(2\pi x_1)))$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i - \sin(2\pi x_1 + \frac{i}{n}\pi))^2$	Linear PF Disconnected PF Nolinear variable linkage
GLT2	$[0,1] \times [-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))(1 - \cos\frac{x_1}{2}\pi)$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(10 - 10\sin(\frac{x_1}{2}\pi))$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i - \sin(2\pi x_1 + \frac{i}{n}\pi))^2$	Convex PF Nolinear variable linkage
GLT3	$[0,1] \times [-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1$ if $f_1(\mathbf{x}) \leq 0.05$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(1 - 19x_1)$ if $f_1(\mathbf{x}) > 0.05$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(\frac{1}{19} - \frac{x_1}{19})$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i - \sin(2\pi x_1 + \frac{i}{n}\pi))^2$	Convex PF Nolinear variable linkage
GLT4	$[0,1] \times [-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(2 - 2x_1^{0.5}\cos^2(2x_1^{0.5}\pi))$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i - \sin(2\pi x_1 + \frac{i}{n}\pi))^2$	Convex PF Disconnected PF Nolinear variable linkage
GLT5	$[0,1]^2 \times [-1,1]^{n-2}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))(1 - \cos(\frac{x_1}{2}\pi))(1 - \cos(\frac{x_2}{2}\pi))$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(1 - \cos(\frac{x_1}{2}\pi))(1 - \sin(\frac{x_2}{2}\pi))$ $f_3(\mathbf{x}) = (1+g(\mathbf{x}))(1 - \sin(\frac{x_1}{2}\pi))$ $g(\mathbf{x}) = \sum_{i=3}^{n}(x_i - \sin(2\pi x_1 + \frac{i}{n}\pi))^2$	Convex PF Nolinear variable linkage
GLT6	$[0,1]^2 \times [-1,1]^{n-2}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))(1 - \cos(\frac{x_1}{2}\pi))(1 - \cos(\frac{x_2}{2}\pi))$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(1 - \cos(\frac{x_1}{2}\pi))(1 - \sin(\frac{x_2}{2}\pi))$ $f_3(\mathbf{x}) = (1+g(\mathbf{x}))(2 - \sin(\frac{x_1}{2}\pi) - \text{sign}(\cos(4x_1\pi)))$ $g(\mathbf{x}) = \sum_{i=3}^{n}(x_i - \sin(2\pi x_1 + \frac{i}{n}\pi))^2$	Convex PF Disconnected PF Nolinear variable linkage

Motivación

- Optimización multi-objetivo: aproximación del frente de Pareto
  - Alta calidad
  - Elevada diversidad
- Multitud de estrategias para preservar la diversidad
- Entre los más conocidos
  - Crowding distance
    - Muy eficiente para dos objetivos
    - Bajo desempeño con tres objetivos
    - Complejidad polinomial
  - Hipervolumen
    - Elevada complejidad, aumenta con el número de objetivos

Contribución

- Nuevo mecanismo de control de diversidad para problemas de tres objetivos
  - Para metaheurísticas basadas en archivo
- Desviación Espacial Propagada (DEP)
  - Basado en la posición de las soluciones en la aproximación al frente de Pareto
  - **Primer método de tiempo polinomial para problemas 3D**

Motivación

- Diversas características del espacio de búsqueda afectan el desempeño de algunos operadores evolutivos.
  - Variables encadenadas
  - Epítasis
  - Rotación

- Un mecanismo de selección que elija el operador más adecuado en función de su desempeño en el proceso de búsqueda, puede mejorar la calidad del frente generado.

Contribución

- Nuevo algoritmo multiobjetivo de estado estable que utiliza desviación espacial propagada y un controlador difuso para seleccionar los operadores disponibles.

Desviación espacial propagada

Desviación Espacial Propagada (DEP)

1. Se fuerza que todas las soluciones extremas sean conservadas, asignándoles aptitud = -∞
2. Se penaliza la aptitud de todas las soluciones con su momento central en $D_{max} - D_{min}$

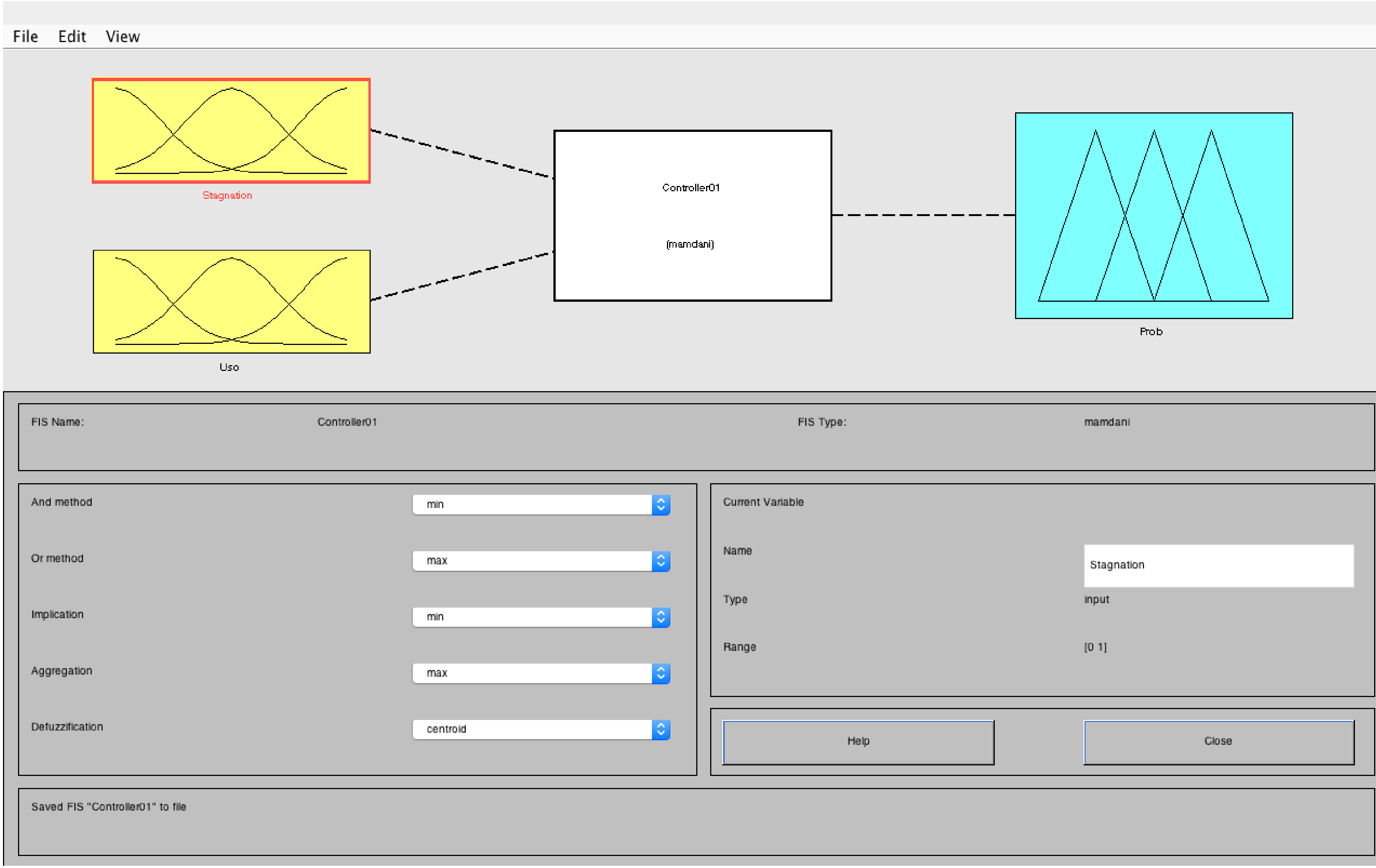
$$\sum_{i=1}^{n} (Dist(i,j) - (D_{max} - D_{min}))^2$$

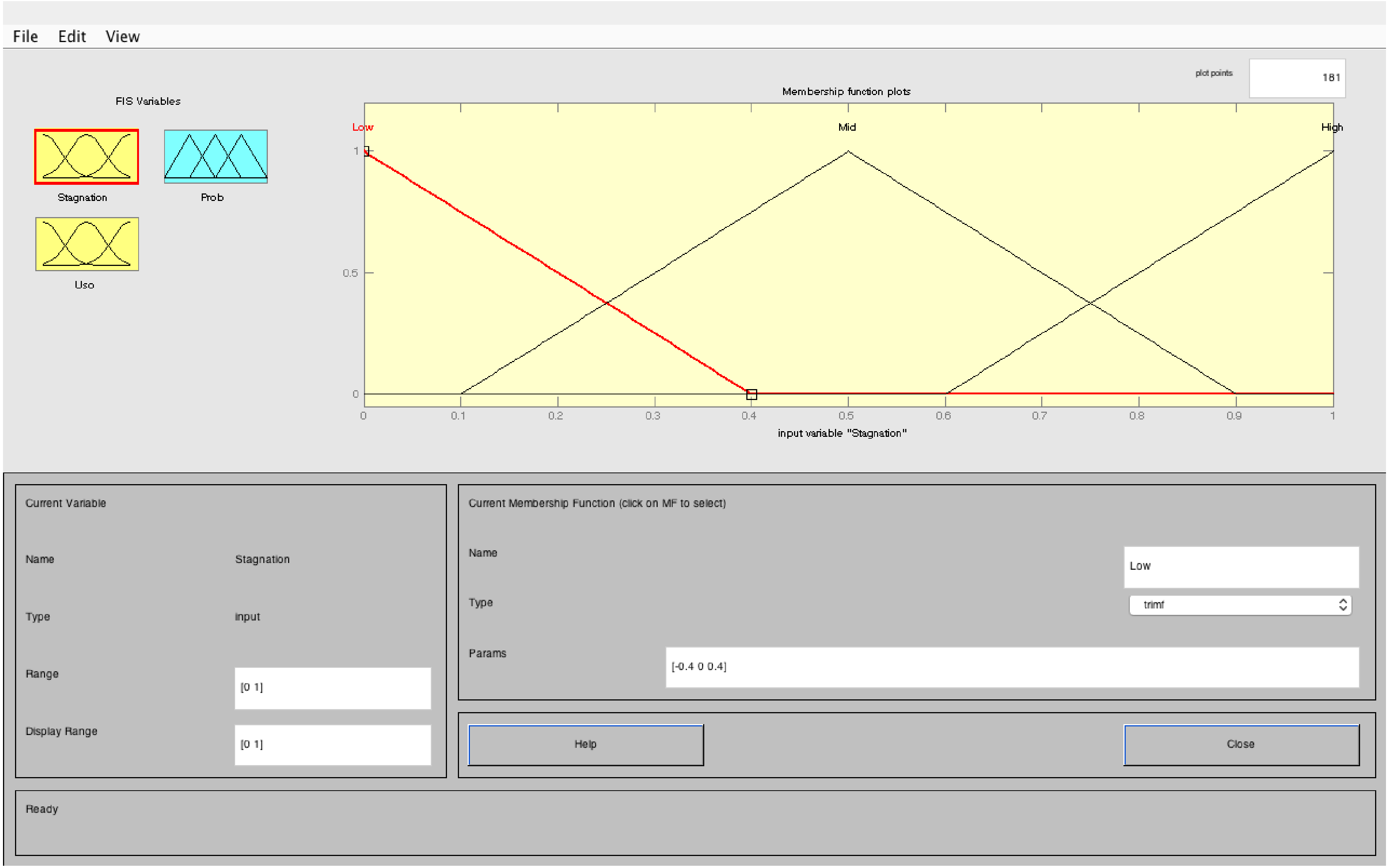
3. Se penaliza la aptitud de las *k* soluciones más cercanas a cada solución

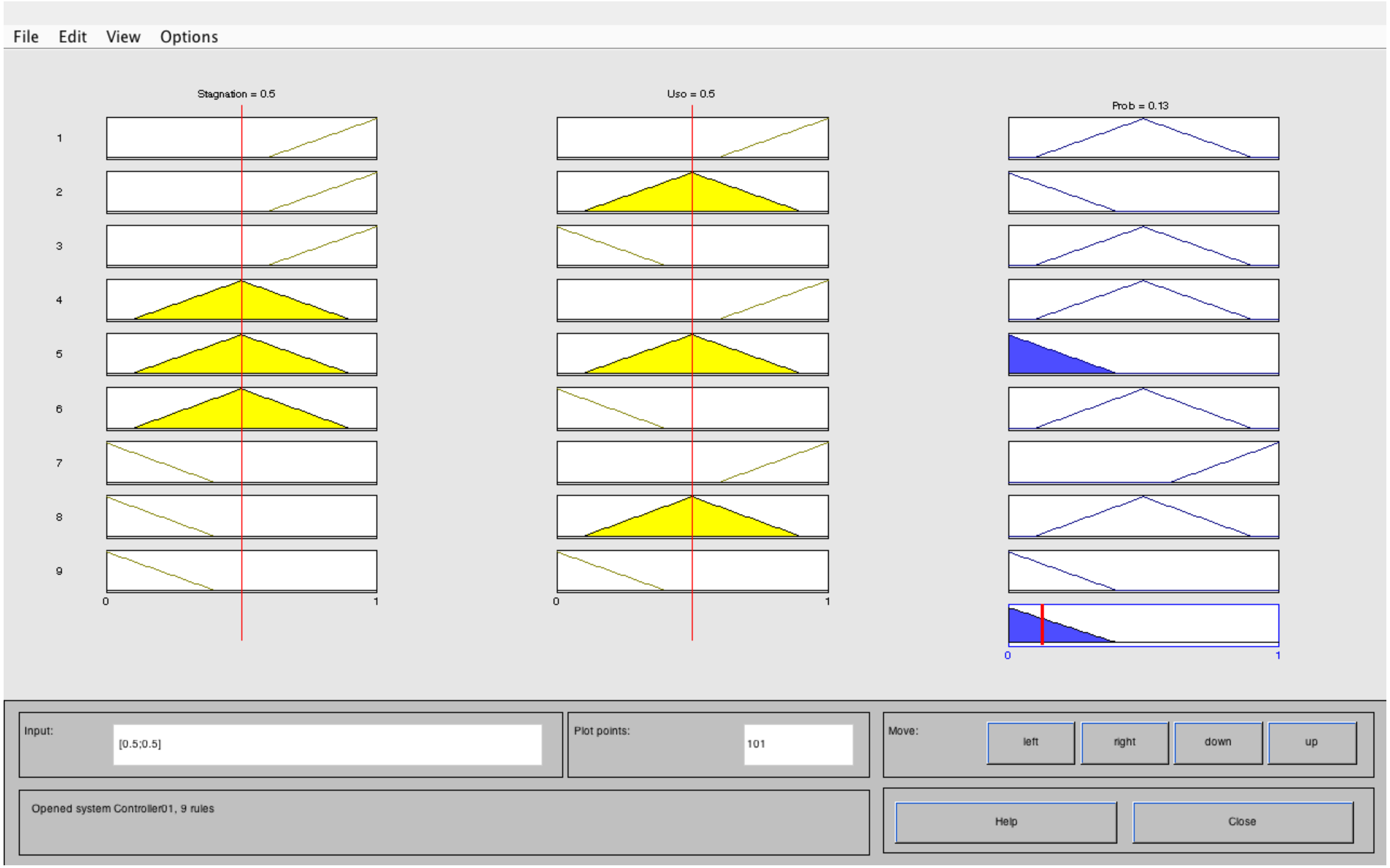
$((D_{max} - D_{min})/(Dist(i,j) + D_{min}))/\ j \in k$ soluciones más próximas a i

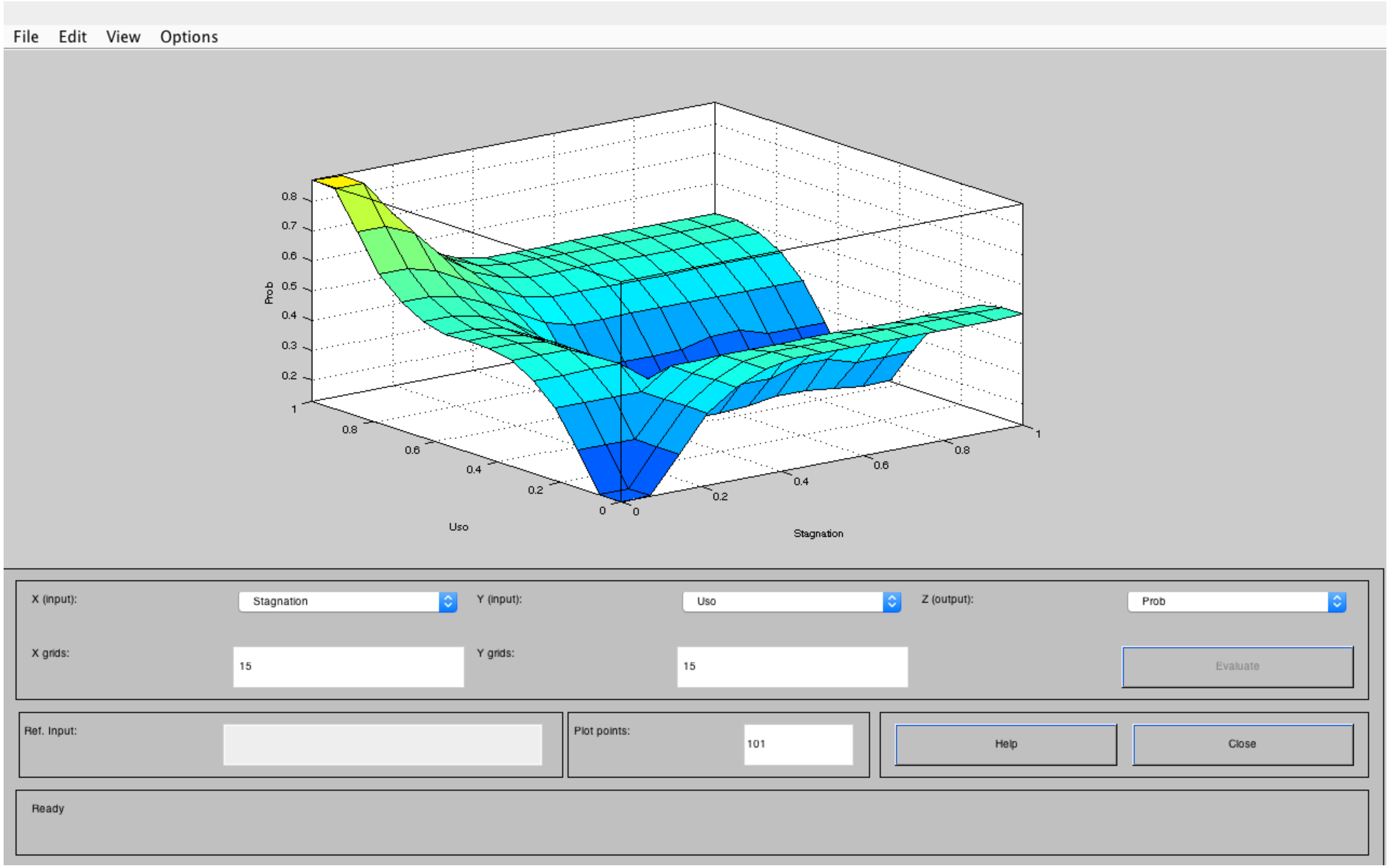
4. Se eligen las soluciones con menor valor de aptitud

Controlador Difuso


File
Edit
View
Stagnation
Uso
Controller01
(mamdani)
Prob
FIS Name:
Controller01
FIS Type:
mamdani
And method
min
Or method
max
Implication
min
Aggregation
max
Defuzzification
centroid
Current Variable
Name
Stagnation
Type
input
Range
[0 1]
Help
Close
Saved FIS "Controller01" to file


File Edit View
FIS Variables
Stagnation
Prob
Uso
plot points
181
Membership function plots
Low
Mid
High
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
input variable "Stagnation"
Current Variable
Name
Stagnation
Type
input
Range
[0 1]
Display Range
[0 1]
Current Membership Function (click on MF to select)
Name
Low
Type
trimf
Params
[-0.4 0 0.4]
Help
Close
Ready


File
Edit
View
Options
Stagnation = 0.5
Uso = 0.5
Prob = 0.13
1
2
3
4
5
6
7
8
9
0
1
0
1
0
1
Input:
[0.5;0.5]
Plot points:
101
Move:
left
right
down
up
Opened system Controller01, 9 rules
Help
Close



File Edit View Options
Prob
0.8
0.7
0.6
0.5
0.4
0.3
0.2
1
0.8
0.6
0.4
0.2
0
Uso
0
0.2
0.4
0.6
0.8
1
Stagnation
X (input):
Stagnation
Y (input):
Uso
Z (output):
Prob
X grids:
15
Y grids:
15
Evaluate
Ref. Input:
Plot points:
101
Help
Close
Ready

Algoritmo FAME

Algorithm 2 Pseudo-code of FAME

Step 1 Initialization

$Initialize(Pop)$

$Archive.add(Pop)$

$SetOperatorsProb(1.0)$

$window = Stagnation = 0$

Step 2 Main loop

while Stopping criterion not reached **do**

  Step 2.1 Parents selection

  for $i = 1$ to $|Parents|$ **do**

    if $RandomDouble(0, 1) \leq \beta$ **then**

      $Parents[i] \leftarrow TournamentSSD(Archive)$

    else

      $Parents[i] \leftarrow TournamentSSD(Pop)$

    end if

  end for

Step 2.2 Reproduction

```
Operator ← Roulette(OpProb)
Offspring ← Operator(Parents)
Evaluate(Offspring)
OpUse = UpdateUse(Operator)
window++
```

Step 2.3 Archive Update

```
if !(Archive.add(Offspring)) then
    Stagnation = UpdateStagnation()
end if
```

Step 2.4 Call to the Fuzzy Inference System

```
if window == windowSize then
    UpdateOpProb(OpUse, Stagnation)
    window = Stagnation = 0
end if
```

Step 2.5 Population Update

```
    NewPop ← Pop.add(Offspring)
    fast_non_dominated_sortSSD(NewPop)
    Pop ← RemoveWorstSolution(NewPop)
end while
```

Step 3 Output

```
return Archive
```

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Resultados

Operadores de recombinación

- Cruza binaria simulada (SBX)
- Evolución diferencial (DE1)
- Mutación Polinomial
- Mutación uniforme

Algoritmos en comparación

- Borg
  - Diseñado para resolver problemas con muchos objetivos.
- SMS-EMOA
  - Aproxima el frente de Pareto maximizando HV.
- SMEA
  - Combina la búsqueda evolutiva con aprendizaje máquina.
- SMPSOhv
  - Algoritmo multi-objetivo de enjambre de partículas.
- MOEA/D
  - Descompone el problema multiobjetivo en un conjunto de problemas mono objetivo.

Instance	Objectives and PSs	Variable Bounds
F1	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2$ where $J_1 = \{j \mid j$ is odd and $2 \le j \le n\}$ and $J_2 = \{j \mid j$ is even and $2 \le j \le n\}$. Its PS is $x_j = x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \dots, n$.	$[0,1]^n$
F2	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2$ where J_1 and J_2 are the same as those of F1. Its PS is $x_j = \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \dots, n$.	$[0,1] \times [-1,1]^{n-1}$
F3	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - 0.8x_1\cos(6\pi x_1 + \frac{j\pi}{n}))^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}))^2$ where J_1 and J_2 are the same as those of F1. Its PS is $x_j = \begin{cases} 0.8x_1\cos(6\pi x_1 + \frac{j\pi}{n}) & j \in J_1 \\ 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}) & j \in J_2 \end{cases}$	$[0,1] \times [-1,1]^{n-1}$
F4	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - 0.8x_1\cos(\frac{6\pi x_1 + \frac{j\pi}{n}}{3}))^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}))^2$ where J_1 and J_2 are the same as those of F1. Its PS is $x_j = \begin{cases} 0.8x_1\cos(\frac{6\pi x_1 + \frac{j\pi}{n}}{3}) & j \in J_1 \\ 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}) & j \in J_2 \end{cases}$	$[0,1] \times [-1,1]^{n-1}$
F5	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}\{x_j - [0.3x_1^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\cos(6\pi x_1 + \frac{j\pi}{n})\}^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}\{x_j - [0.3x_1^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\sin(6\pi x_1 + \frac{j\pi}{n})\}^2$ where J_1 and J_2 are the same as those of F1.	$[0,1] \times [-1,1]^{n-1}$

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	Its PS is $x_j = \begin{cases} [0.3x_1^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\cos(6\pi x_1 + \frac{j\pi}{n}) & j \in J_1 \\ [0.3x_1^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\sin(6\pi x_1 + \frac{j\pi}{n}) & j \in J_2 \end{cases}$	
F6	$f_1 = \cos(0.5x_1\pi)\cos(0.5x_2\pi) + \frac{2}{\vert J_1\vert}\sum_{j\in J_1}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2$ $f_2 = \cos(0.5x_1\pi)\sin(0.5x_2\pi) + \frac{2}{\vert J_2\vert}\sum_{j\in J_2}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2$ $f_3 = \sin(0.5x_1\pi) + \frac{2}{\vert J_3\vert}\sum_{j\in J_3}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2$ where $J_1 = \{j \vert 3 \le j \le n$, and $j-1$ is a multiplication of $3\}$, $J_2 = \{j \vert 3 \le j \le n$, and $j-2$ is a multiplication of $3\}$, $J_3 = \{j \vert 3 \le j \le n$, and j is a multiplication of $3\}$ Its PS is $x_j = 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}), j = 3, \ldots, n$.	$[0,1]^2 \times [-2,2]^{n-2}$
F7	$f_1 = x_1 + \frac{2}{\vert J_1\vert}\sum_{j\in J_1}(4y_j^2 - \cos(8y_j\pi) + 1.0)$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\vert J_2\vert}\sum_{j\in J_2}(4y_j^2 - \cos(8y_j\pi) + 1.0)$ where J_1 and J_2 are the same as those of F1 and $y_j = x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \ldots, n$. Its PS is $x_j = x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \ldots, n$.	$[0,1]^n$
F8	$f_1 = x_1 + \frac{2}{\vert J_1\vert}(4\sum_{j\in J_1} y_j^2 - 2\prod_{j\in J_1}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{\vert J_2\vert}(4\sum_{j\in J_2} y_j^2 - 2\prod_{j\in J_2}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$ where J_1 and J_2 are the same as those of F1 and $y_j = x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \ldots, n$. Its PS is $x_j = x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \ldots, n$.	$[0,1]^n$
F9	$f_1 = x_1 + \frac{2}{\vert J_1\vert}\sum_{j\in J_1}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2$ $f_2 = 1 - x_1^2 + \frac{2}{\vert J_2\vert}\sum_{j\in J_2}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2$ where J_1 and J_2 are the same as those of F1. Its PS is $x_j = \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n$.	$[0,1] \times [-1,1]^{n-1}$

Instance	Variable Space	Objective Function	Characteristic
GLT1	$[0,1]\times[-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(2-x_1-\text{sign}(\cos(2\pi x_1)))$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i-\sin(2\pi x_1+\frac{i}{n}\pi))^2$	Linear PF Disconnected PF Nolinear variable linkage
GLT2	$[0,1]\times[-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))(1-\cos\frac{x_1}{2}\pi)$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(10-10\sin(\frac{x_1}{2}\pi))$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i-\sin(2\pi x_1+\frac{i}{n}\pi))^2$	Convex PF Nolinear variable linkage
GLT3	$[0,1]\times[-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1$ if $f_1(\mathbf{x}) \leq 0.05$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(1-19x_1)$ if $f_1(\mathbf{x}) > 0.05$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(\frac{1}{19}-\frac{x_1}{19})$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i-\sin(2\pi x_1+\frac{i}{n}\pi))^2$	Convex PF Nolinear variable linkage
GLT4	$[0,1]\times[-1,1]^{n-1}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))x_1$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(2-2x_1^{0.5}\cos^2(2x_1^{0.5}\pi))$ $g(\mathbf{x}) = \sum_{i=2}^{n}(x_i-\sin(2\pi x_1+\frac{i}{n}\pi))^2$	Convex PF Disconnected PF Nolinear variable linkage
GLT5	$[0,1]^2\times[-1,1]^{n-2}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))(1-\cos(\frac{x_1}{2}\pi))(1-\cos(\frac{x_2}{2}\pi))$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(1-\cos(\frac{x_1}{2}\pi))(1-\sin(\frac{x_2}{2}\pi))$ $f_3(\mathbf{x}) = (1+g(\mathbf{x}))(1-\sin(\frac{x_1}{2}\pi))$ $g(\mathbf{x}) = \sum_{i=3}^{n}(x_i-\sin(2\pi x_1+\frac{i}{n}\pi))^2$	Convex PF Nolinear variable linkage
GLT6	$[0,1]^2\times[-1,1]^{n-2}$	$f_1(\mathbf{x}) = (1+g(\mathbf{x}))(1-\cos(\frac{x_1}{2}\pi))(1-\cos(\frac{x_2}{2}\pi))$ $f_2(\mathbf{x}) = (1+g(\mathbf{x}))(1-\cos(\frac{x_1}{2}\pi))(1-\sin(\frac{x_2}{2}\pi))$ $f_3(\mathbf{x}) = (1+g(\mathbf{x}))(2-\sin(\frac{x_1}{2}\pi)-\text{sign}(\cos(4x_1\pi)))$ $g(\mathbf{x}) = \sum_{i=3}^{n}(x_i-\sin(2\pi x_1+\frac{i}{n}\pi))^2$	Convex PF Disconnected PF Nolinear variable linkage

Problem	BORG	MOEA/D-DE	SMEA	SMPSOhv	SMS-EMOA	FAME
			I_{HV}			
LZ09F1	$6.59E-1_{1.3E-4}$▲	$6.61E-1_{1.5E-4}$▲	$6.56E-1_{5.5E-4}$▲	$6.60E-1_{6.2E-4}$▲	$6.42E-1_{8.1E-3}$▲	$6.61E-1_{1.8E-4}$
LZ09F2	$5.36E-1_{2.4E-2}$▲	$6.52E-1_{2.1E-3}$▽	$6.26E-1_{2.7E-2}$▲	$5.43E-1_{2.3E-2}$▲	$5.36E-1_{3.0E-2}$▲	$6.46E-1_{2.7E-3}$
LZ09F3	$2.88E-1_{1.5E-2}$▲	$6.47E-1_{3.1E-2}-$	$6.39E-1_{6.3E-3}$▲	$6.26E-1_{6.6E-3}$▲	$6.16E-1_{1.2E-2}$▲	$6.46E-1_{2.4E-3}$
LZ09F4	$6.25E-1_{6.5E-3}$▲	$6.49E-1_{2.7E-3}$▽	$6.37E-1_{4.4E-3}$▲	$6.26E-1_{6.7E-3}$▲	$6.30E-1_{5.0E-3}$▲	$6.48E-1_{1.5E-3}$
LZ09F5	$6.40E-1_{4.8E-3}$▲	$6.40E-1_{5.7E-3}-$	$6.42E-1_{2.1E-3}-$	$6.32E-1_{4.3E-3}$▲	$6.27E-1_{8.9E-3}$▲	$6.42E-1_{6.2E-3}$
LZ09F6	$2.08E-1_{5.3E-6}$▲	$2.42E-1_{7.1E-2}$▲	$2.76E-1_{2.8E-2}$▲	$3.90E-1_{1.3E-2}$▽	$3.44E-1_{2.5E-2}$▲	$3.45E-1_{4.3E-2}$
LZ09F7	$6.14E-1_{1.3E-1}$▲	$6.55E-1_{3.2E-3}$▲	$6.32E-1_{1.8E-2}$▲	$5.07E-1_{1.3E-1}$▲	$4.41E-1_{9.4E-2}$▲	$6.56E-1_{8.7E-4}$
LZ09F8	$4.06E-1_{1.0E-1}$▲	$5.50E-1_{6.4E-2}$▲	$4.72E-1_{7.2E-2}$▲	$3.57E-1_{2.0E-1}$▲	$4.38E-1_{7.2E-2}$▲	$5.60E-1_{2.7E-2}$
LZ09F9	$2.17E-1_{4.6E-2}$▲	$3.17E-1_{5.0E-3}$▽	$2.73E-1_{2.8E-2}$▲	$2.15E-1_{4.0E-2}$▲	$2.01E-1_{6.3E-2}$▲	$3.13E-1_{2.6E-3}$
GLT1	$1.01E-2_{4.5E-2}$▲	$3.72E-1_{2.2E-4}$▽	$3.48E-1_{2.0E-2}$▲	$1.75E-1_{5.8E-2}$▲	$2.97E-2_{2.9E-2}$▲	$3.72E-1_{8.6E-4}$
GLT2	$7.25E-1_{9.7E-2}$▲	$7.53E-1_{5.6E-3}$▲	$7.76E-1_{6.2E-4}$▲	$7.66E-1_{3.0E-3}$▲	$6.46E-1_{1.1E-1}$▲	$7.79E-1_{2.7E-4}$
GLT3	$9.35E-1_{6.6E-3}$▲	$9.44E-1_{1.3E-3}-$	$9.46E-1_{3.8E-3}-$	$9.39E-1_{5.1E-3}$▲	$9.26E-1_{8.3E-3}$▲	$9.44E-1_{4.4E-3}$
GLT4	$4.30E-1_{3.3E-1}$▲	$4.94E-1_{3.5E-4}$▽	$4.91E-1_{2.1E-2}-$	$4.32E-1_{2.3E-2}-$	$3.45E-1_{2.9E-1}$▲	$4.62E-1_{2.4E-1}$
GLT5	$9.58E-1_{4.2E-3}$▲	$9.51E-1_{7.7E-4}$▲	$9.53E-1_{2.1E-3}$▲	$9.69E-1_{4.4E-4}-$	$9.68E-1_{4.6E-3}$▲	$9.69E-1_{3.7E-4}$
GLT6	$9.41E-1_{4.2E-3}$▲	$9.47E-1_{1.1E-3}$▲	$9.45E-1_{1.8E-3}$▲	$9.55E-1_{2.4E-3}$▲	$9.58E-1_{4.3E-3}$▲	$9.62E-1_{5.1E-4}$
			$I_{\epsilon+}$			
LZ09F1	$9.91E-3_{1.8E-4}$▲	$9.73E-3_{1.8E-3}$▲	$1.04E-2_{1.4E-3}$▲	$7.55E-3_{6.9E-4}$▲	$1.43E-1_{3.2E-2}$▲	$6.26E-3_{8.9E-4}$

			$I_{\epsilon+}$			
LZ09F1	$9.91E-3_{1.8E-4}$▲	$9.73E-3_{1.8E-3}$▲	$1.04E-2_{1.4E-3}$▲	$7.55E-3_{6.9E-4}$▲	$1.43E-1_{3.2E-2}$▲	$6.26E-3_{8.9E-4}$
LZ09F2	$1.80E-1_{2.7E-2}$▲	$2.08E-2_{1.1E-2}$▽	$7.22E-2_{6.4E-2}$▲	$1.31E-1_{2.5E-2}$▲	$1.88E-1_{6.1E-2}$▲	$2.22E-2_{9.3E-3}$
LZ09F3	$3.01E-1_{2.8E-2}$▲	$6.62E-2_{1.6E-1}$▲	$6.01E-2_{2.6E-2}$▲	$7.92E-2_{1.5E-2}$▲	$1.33E-1_{6.8E-2}$▲	$4.37E-2_{1.5E-2}$
LZ09F4	$1.90E-1_{2.8E-2}$▲	$6.46E-2_{1.0E-2}$▲	$6.96E-2_{2.2E-2}$▲	$8.93E-2_{2.1E-2}$▲	$1.67E-1_{1.8E-2}$▲	$3.54E-2_{8.4E-3}$
LZ09F5	$7.16E-2_{1.8E-2}$▲	$8.66E-2_{1.8E-2}$▲	$4.81E-2_{1.2E-2}$▽	$7.95E-2_{1.2E-2}$▲	$1.21E-1_{4.8E-2}$▲	$6.74E-2_{3.0E-2}$
LZ09F6	$7.07E-1_{3.6E-5}$▲	$2.75E-1_{3.2E-1}$▲	$2.57E-1_{2.9E-2}$▲	$1.55E-1_{1.7E-2}$▽	$2.47E-1_{2.4E-2}$▲	$2.04E-1_{9.6E-2}$
LZ09F7	$1.68E-1_{2.1E-1}$▲	$4.20E-2_{1.4E-2}$▲	$8.10E-2_{4.5E-2}$▲	$2.70E-1_{1.3E-1}$▲	$3.77E-1_{1.5E-1}$▲	$1.19E-2_{2.0E-3}$
LZ09F8	$4.60E-1_{1.4E-1}$▲	$2.15E-1_{1.1E-1}$▲	$2.97E-1_{1.1E-1}$▲	$3.88E-1_{8.5E-2}$▲	$3.92E-1_{1.9E-1}$▲	$1.60E-1_{8.1E-2}$
LZ09F9	$1.89E-1_{9.5E-2}$▲	$2.64E-2_{1.8E-2}$—	$1.15E-1_{3.8E-2}$▲	$1.43E-1_{7.8E-2}$▲	$2.32E-1_{1.1E-1}$▲	$2.45E-2_{1.4E-2}$
GLT1	$2.45E-1_{2.8E-4}$▲	$9.46E-3_{1.5E-3}$▲	$3.53E-2_{3.4E-2}$▲	$2.43E-1_{5.6E-2}$▲	$2.43E-1_{5.7E-3}$▲	$4.88E-3_{5.2E-4}$
GLT2	$1.52E-1_{1.8E-1}$▲	$7.36E-2_{8.1E-3}$▲	$1.25E-2_{2.0E-3}$▲	$2.23E-2_{5.2E-3}$▲	$2.13E-1_{1.5E-1}$▲	$9.66E-3_{1.2E-3}$
GLT3	$2.78E-2_{1.4E-2}$▲	$1.67E-2_{6.5E-3}$▽	$1.58E-2_{1.2E-2}$▽	$2.57E-2_{1.1E-2}$▲	$4.73E-2_{1.3E-2}$▲	$2.23E-2_{1.1E-2}$
GLT4	$1.62E-1_{2.9E-1}$▲	$1.33E-2_{2.9E-3}$▽	$1.75E-2_{8.7E-2}$—	$1.70E-1_{8.3E-2}$—	$1.85E-1_{2.8E-1}$▲	$1.19E-1_{4.0E-1}$
GLT5	$4.74E-2_{2.6E-2}$▲	$5.05E-2_{1.7E-3}$▲	$8.33E-2_{9.3E-3}$▲	$3.16E-2_{2.7E-3}$▽	$5.49E-2_{3.2E-2}$▲	$3.47E-2_{3.1E-3}$
GLT6	$6.04E-2_{1.0E-2}$▲	$5.23E-2_{2.2E-3}$▲	$7.35E-2_{7.3E-3}$▲	$5.56E-2_{3.3E-3}$▲	$5.18E-2_{3.6E-3}$▲	$3.70E-2_{2.8E-2}$
			I_{GS}			
LZ09F1	$5.67E-1_{5.5E-3}$▲	$5.46E-1_{4.9E-3}$▲	$2.78E-1_{1.5E-2}$▽	$3.91E-1_{4.2E-2}$▽	$1.26E0_{8.4E-2}$▲	$4.74E-1_{8.6E-3}$
LZ09F2	$5.79E-1_{8.0E-2}$▲	$4.27E-1_{1.0E-1}$▲	$5.76E-1_{3.1E-1}$▲	$2.84E-1_{1.0E-1}$—	$5.98E-1_{5.2E-2}$▲	$2.62E-1_{5.3E-2}$

	I_{GS}					
LZ09F1	$5.67E-1_{5.5E-3}\blacktriangle$	$5.46E-1_{4.9E-3}\blacktriangle$	$2.78E-1_{1.5E-2}\triangledown$	$3.91E-1_{4.2E-2}\triangledown$	$1.26E0_{8.4E-2}\blacktriangle$	$4.74E-1_{8.6E-3}$
LZ09F2	$5.79E-1_{8.0E-2}\blacktriangle$	$4.27E-1_{1.0E-1}\blacktriangle$	$5.76E-1_{3.1E-1}\blacktriangle$	$2.84E-1_{1.0E-1}-$	$5.98E-1_{5.2E-2}\blacktriangle$	$2.62E-1_{5.3E-2}$
LZ09F3	$3.47E-1_{3.5E-2}\triangledown$	$8.58E-1_{3.0E-1}\blacktriangle$	$5.49E-1_{1.5E-1}\blacktriangle$	$6.81E-1_{3.2E-2}\blacktriangle$	$1.01E0_{9.7E-2}\blacktriangle$	$4.28E-1_{1.2E-1}$
LZ09F4	$1.24E0_{2.8E-2}\blacktriangle$	$9.31E-1_{6.5E-2}\blacktriangle$	$5.99E-1_{1.5E-1}\blacktriangle$	$6.50E-1_{6.1E-2}\blacktriangle$	$1.13E0_{8.0E-2}\blacktriangle$	$3.62E-1_{5.5E-2}$
LZ09F5	$8.97E-1_{1.0E-1}\blacktriangle$	$9.23E-1_{1.0E-1}\blacktriangle$	$5.22E-1_{8.6E-2}\triangledown$	$7.50E-1_{3.0E-2}\blacktriangle$	$1.07E0_{1.1E-1}\blacktriangle$	$6.20E-1_{2.2E-1}$
LZ09F6	$6.46E-1_{4.6E-5}\blacktriangle$	$6.14E-1_{2.0E-1}\blacktriangle$	$4.61E-1_{8.0E-2}\blacktriangle$	$5.97E-1_{7.3E-2}\blacktriangle$	$7.57E-1_{1.0E-1}\blacktriangle$	$4.24E-1_{1.3E-1}$
LZ09F7	$1.06E0_{4.2E-1}\blacktriangle$	$7.51E-1_{1.5E-1}\blacktriangle$	$7.98E-1_{2.6E-1}\blacktriangle$	$9.84E-1_{3.2E-1}\blacktriangle$	$5.85E-1_{9.6E-2}\blacktriangle$	$3.56E-1_{4.0E-2}$
LZ09F8	$6.08E-1_{1.2E-1}\blacktriangle$	$8.10E-1_{2.7E-1}\blacktriangle$	$8.02E-1_{1.5E-1}\blacktriangle$	$7.92E-1_{4.9E-1}\blacktriangle$	$5.81E-1_{1.1E-1}\blacktriangle$	$5.09E-1_{1.9E-1}$
LZ09F9	$5.93E-1_{9.7E-2}\blacktriangle$	$4.21E-1_{1.3E-1}\blacktriangle$	$7.42E-1_{8.7E-2}\blacktriangle$	$4.38E-1_{2.4E-1}\blacktriangle$	$6.02E-1_{5.1E-2}\blacktriangle$	$3.32E-1_{8.8E-2}$
GLT1	$5.21E-1_{9.7E-2}\blacktriangle$	$4.74E-1_{1.1E-2}\blacktriangle$	$5.84E-1_{4.5E-1}\blacktriangle$	$9.45E-1_{1.6E-1}\blacktriangle$	$5.64E-1_{6.5E-2}\blacktriangle$	$4.12E-1_{1.4E-2}$
GLT2	$9.17E-1_{1.4E-1}\blacktriangle$	$9.85E-1_{1.2E-1}\blacktriangle$	$3.68E-1_{2.5E-2}\triangledown$	$3.31E-1_{6.5E-2}\triangledown$	$7.97E-1_{1.8E-1}\blacktriangle$	$4.47E-1_{1.3E-2}$
GLT3	$6.32E-1_{1.5E-1}\triangledown$	$9.39E-1_{1.3E-1}\triangledown$	$1.21E0_{4.7E-1}\triangledown$	$1.16E0_{8.9E-2}\triangledown$	$1.11E0_{1.2E-2}\triangledown$	$1.34E0_{9.2E-2}$
GLT4	$1.18E0_{6.9E-1}\blacktriangle$	$7.25E-1_{1.6E-2}\triangledown$	$4.98E-1_{8.5E-1}\triangledown$	$1.22E0_{2.1E-1}\blacktriangle$	$6.63E-1_{3.4E-1}\triangledown$	$9.24E-1_{4.6E-1}$
GLT5	$6.16E-1_{1.4E-1}\blacktriangle$	$7.92E-1_{6.1E-3}\blacktriangle$	$3.46E-1_{2.2E-2}\blacktriangle$	$3.47E-1_{1.5E-2}\blacktriangle$	$5.84E-1_{2.1E-1}\blacktriangle$	$3.12E-1_{8.4E-3}$
GLT6	$6.70E-1_{9.9E-2}\blacktriangle$	$7.83E-1_{8.6E-3}\blacktriangle$	$3.99E-1_{2.7E-2}\blacktriangle$	$3.98E-1_{6.6E-2}\blacktriangle$	$4.70E-1_{1.1E-1}\blacktriangle$	$3.73E-1_{1.8E-2}$

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Table 4: Average rankings of the algorithms

I_{HV}		$I_{\epsilon+}$		I_{GS}	
Algorithm	Rank	Algorithm	Rank	Algorithm	Rank
FAME	1.78	FAME	1.75	FAME	2.09
MOEA/D-DE	2.57	MOEA/D-DE	2.85	SMEA	3.02
SMEA	3.32	SMEA	3.22	SMPSOhv	3.21
SMPSOhv	3.90	SMPSOhv	3.52	MOEA/D-DE	4.11
SMS-EMOA	4.63	BORG	4.77	BORG	4.16
BORG	4.79	SMS-EMOA	4.90	SMS-EMOA	4.42

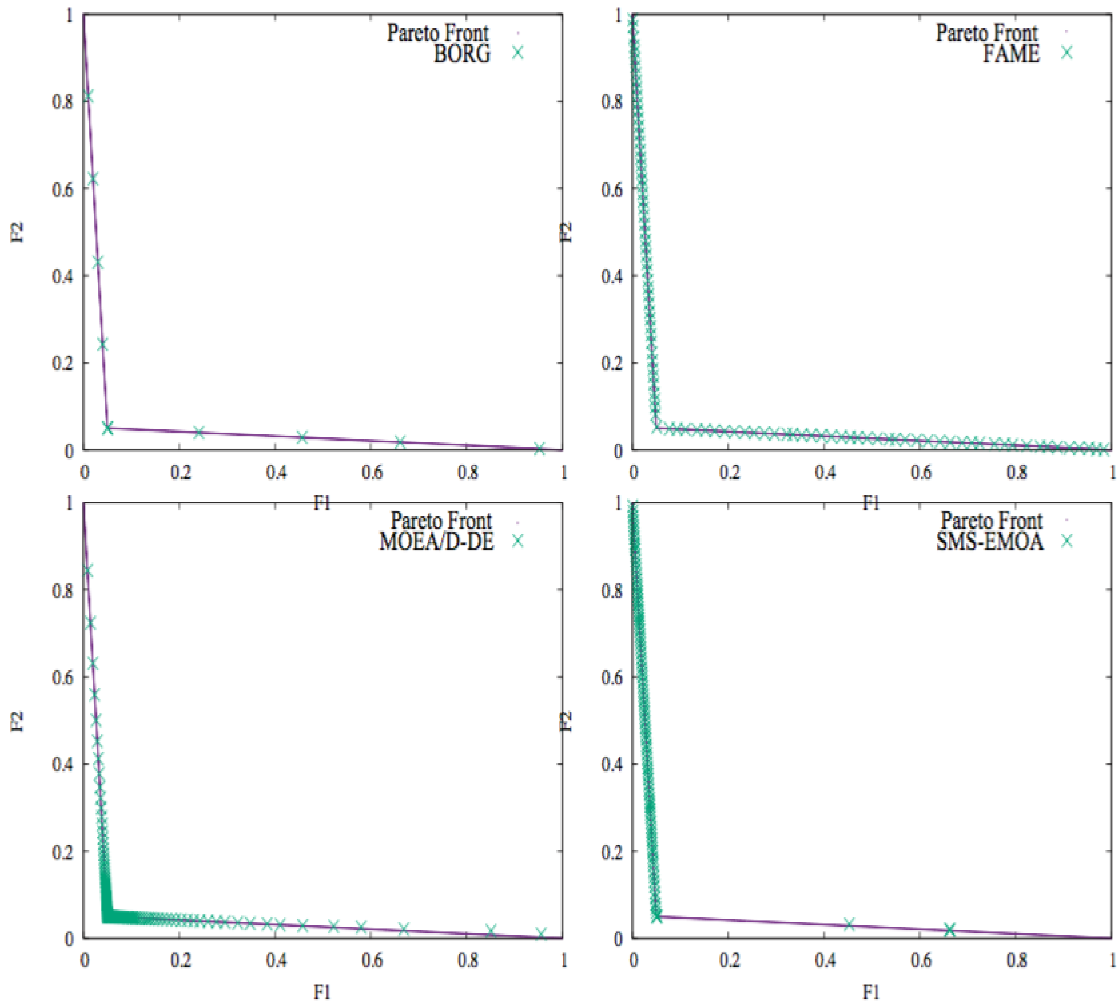

Figure 5: Best Pareto fronts reported by the three best performing algorithms, as well as FAME, for GLT3 problem according to I_{GS}.

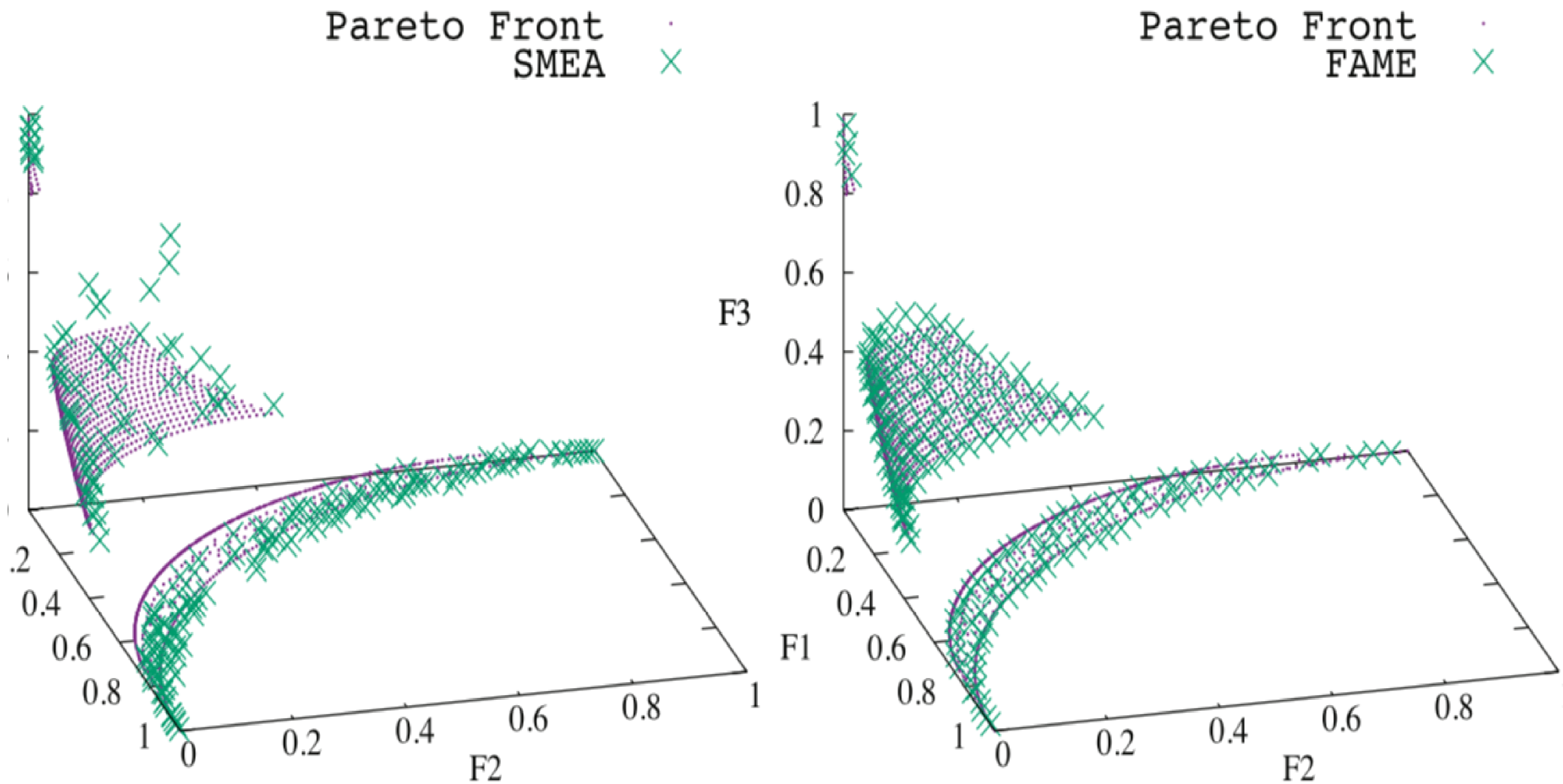

Figure 6: Best Pareto fronts reported by SMEA and FAME for GLT6 according to I_{GS}.

	I_{HV}		$I_{\epsilon+}$		I_{GS}	
Problem	FAME-CD	FAME	FAME-CD	FAME	FAME-CD	FAME
LZ09F1	$6.62E-1_{8.3E-5}\triangledown$	$6.61E-1_{1.8E-4}$	$5.53E-3_{2.6E-4}\triangledown$	$6.26E-3_{8.9E-4}$	$4.89E-1_{6.4E-3}\blacktriangle$	$4.74E-1_{8.6E-3}$
LZ09F2	$6.47E-1_{3.7E-3}-$	$6.46E-1_{2.7E-3}$	$2.41E-2_{1.6E-2}-$	$2.22E-2_{9.3E-3}$	$2.93E-1_{1.0E-1}\blacktriangle$	$2.62E-1_{5.3E-2}$
LZ09F3	$6.48E-1_{2.4E-3}\triangledown$	$6.46E-1_{2.4E-3}$	$4.17E-2_{1.7E-2}-$	$4.37E-2_{1.5E-2}$	$4.68E-1_{1.3E-1}-$	$4.28E-1_{1.2E-1}$
LZ09F4	$6.49E-1_{1.8E-3}\triangledown$	$6.48E-1_{1.5E-3}$	$3.75E-2_{1.0E-2}-$	$3.54E-2_{8.4E-3}$	$4.34E-1_{9.0E-2}\blacktriangle$	$3.62E-1_{5.5E-2}$
LZ09F5	$6.44E-1_{7.1E-3}-$	$6.42E-1_{6.2E-3}$	$6.83E-2_{3.0E-2}-$	$6.74E-2_{3.0E-2}$	$6.99E-1_{2.3E-1}\blacktriangle$	$6.20E-1_{2.2E-1}$
LZ09F6	$2.94E-1_{3.0E-2}\blacktriangle$	$3.45E-1_{4.3E-2}$	$2.47E-1_{6.0E-2}\blacktriangle$	$2.04E-1_{9.6E-2}$	$4.80E-1_{6.7E-2}\blacktriangle$	$4.24E-1_{1.3E-1}$
LZ09F7	$6.56E-1_{1.2E-3}-$	$6.56E-1_{8.7E-4}$	$1.19E-2_{2.2E-3}-$	$1.19E-2_{2.0E-3}$	$3.41E-1_{3.7E-2}\triangledown$	$3.56E-1_{4.0E-2}$
LZ09F8	$5.56E-1_{2.9E-2}-$	$5.60E-1_{2.7E-2}$	$1.56E-1_{9.8E-2}-$	$1.60E-1_{8.1E-2}$	$5.10E-1_{2.1E-1}-$	$5.09E-1_{1.9E-1}$
LZ09F9	$3.13E-1_{4.1E-3}\blacktriangle$	$3.13E-1_{2.6E-3}$	$2.68E-2_{1.6E-2}-$	$2.45E-2_{1.4E-2}$	$3.56E-1_{1.2E-1}-$	$3.32E-1_{8.8E-2}$
GLT1	$3.72E-1_{8.9E-4}-$	$3.72E-1_{8.6E-4}$	$4.44E-3_{6.0E-4}\triangledown$	$4.88E-3_{5.2E-4}$	$4.11E-1_{1.1E-2}-$	$4.12E-1_{1.4E-2}$
GLT2	$7.80E-1_{2.5E-4}\triangledown$	$7.79E-1_{2.7E-4}$	$8.36E-3_{1.3E-3}\triangledown$	$9.66E-3_{1.2E-3}$	$4.56E-1_{9.9E-3}\blacktriangle$	$4.47E-1_{1.3E-2}$
GLT3	$9.44E-1_{4.2E-3}-$	$9.44E-1_{4.4E-3}$	$2.17E-2_{1.0E-2}-$	$2.23E-2_{1.1E-2}$	$1.34E0_{1.0E-1}-$	$1.34E0_{9.2E-2}$
GLT4	$2.52E-1_{2.4E-1}-$	$4.62E-1_{2.4E-1}$	$4.08E-1_{4.0E-1}-$	$1.19E-1_{4.0E-1}$	$9.23E-1_{4.7E-1}-$	$9.24E-1_{4.6E-1}$
GLT5	$9.57E-1_{1.4E-3}\blacktriangle$	$9.69E-1_{3.7E-4}$	$8.16E-2_{1.2E-2}\blacktriangle$	$3.47E-2_{3.1E-3}$	$4.28E-1_{3.3E-2}\blacktriangle$	$3.12E-1_{8.4E-3}$
GLT6	$9.47E-1_{2.5E-3}\blacktriangle$	$9.62E-1_{5.1E-4}$	$7.17E-2_{1.2E-2}\blacktriangle$	$3.70E-2_{2.8E-2}$	$4.83E-1_{3.6E-2}\blacktriangle$	$3.73E-1_{1.8E-2}$

We compare now the SSD and CD density estimators. For that, we configure FAME with SSD

Conclusiones

- Dispersión Espacial Propagada (DEP)
  - Nuevo algoritmo para la preservación de la diversidad de soluciones en el frente para problemas 3D
- FAME
  - Nuevo algoritmo multiobjetivo que incorpora DEP y selección difusa de operadores
- Experimentación
  - Comparación de FAME con 5 algoritmos del estado del arte
  - 15 problemas conocidos (LZ & GLT)
  - Tres métricas de calidad
- Resultados
  - El resultado de la prueba de Friedman muestra que el desempeño de FAME **supera significativamente** al de los algoritmos con los que se realizó la comparación,
    - Sobre las tres métricas estudiadas
    - Resultados corroborados visualmente

Áreas abiertas

- Incorporar controladores difusos tipo 2
- Configuración automática de las reglas y funciones de membresía del controlador difuso.
- Incorporar DEP a metahuristicas que utilizan Crowding Distance.
- Optimizar problemas multiobjetivo discretos
- Optimizar problemas multiobjetivo dinámicos, con restricciones y/o con preferencias.
- Optimizar problemas multiobjetivo continuos con con más de 3 objetivos.
- Optimizar problemas de aplicación real.

Referencia

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Algorithm 1 Computation of the Spatial Spread Deviation

```
function Compute_SDD(A)
    n = |A|
    for i ∈ A do
        A[i].SSD = 0
    end for
    for m = 1 to k do
        Sort(A, m)
        A[1].SSD = A[n].SSD = −∞
    end for
    Compute the matrix of normalized distances M
    Find D_max and D_min between solutions i and j, i ≠ j
    for i ∈ A do
        temp = 0
        for ∀j ≠ i ∈ Archive do
            temp = temp + (M[i][j] − (D_max − D_min))^2
        end for
        temp = temp / (n−1)
        [i].SSD = A[i].SSD + √temp
    end for
    for i ∈ A do
        Sort(M[i])
        temp = 0
        for j = 2 to k+1 do
            temp = temp + (D_max − D_min)/(M[i][j])
        end for
        A[i].SSD = A[i].SSD + temp
    end for
end function
```

1) *SBX_Crossover* Algoritmo: *NSGA-II*. (Deb, 2007) $s_1 = \frac{1}{2}(p_1 + p_2) - \frac{1}{2}\beta(p_2 - p_1)$ $s_2 = \frac{1}{2}(p_1 + p_2) + \frac{1}{2}\beta(p_2 - p_1)$	3) *Polynomial Mutation* Algoritmo: *NSGA-II*. (Deb, 2007) $p' = \begin{cases} p + \bar{\delta}_L(p - x_i{}^{(L)}), & for\ u \leq 0.5 \\ p + \bar{\delta}_R(x_i{}^{(U)} - p), & for\ u > 0.5 \end{cases}$
2) *DE_Crossover.* Algoritmo: *DE*. (Storn & Price, 1997) $s = p_c + f(p_a - p_b)$	4) *Uniform Mutation* Algoritmo: *OMOPSO*. (Sierra & Coello, 2005) $x_i = \begin{cases} x_i, for\ u \leq 0.5 \\ x_i', \quad for\ u > 0.5 \end{cases}$ $x_i' \in [L_i, U_i]$